Using the J1-J2 Quantum Spin Chain as an Adiabatic Quantum Data Bus

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Abstract

This paper investigates numerically a phenomenon which can be used to transport a single q-bit down a J1-J2 Heisenberg spin chain using a quantum adiabatic process. The motivation for investigating such processes comes from the idea that this method of transport could potentially be used as a means of sending data to various parts of a quantum computer made of artificial spins, and that this method could take advantage of the easily prepared ground state at the so called Majumdar-Ghosh point. We examine several annealing protocols for this process and find similar results for all of them. The annealing process works well up to a critical frustration threshold. There is also a brief section examining what other models this protocol could be used for, examining its use in the XXZ and XYZ models.

Introduction

The ability to send data from one part of a computer to another accurately and quickly is an essential feature in virtually any design. The use of artificial spin clusters in quantum computing has been of growing interest. There is an implementation which has been demonstrated using superconducting flux q-bits[1, 2, 3, 4, 5]. This paper demonstrates an effective and scalable way of sending arbitrary q-bit states along a spin chain with Heisenberg type coupling using quantum annealing. Assuming one could implement a Hamiltonian which follows this model, for example using the methods proposed in [6] using coupled cavities, this system design could be used for a data bus which transports quantum states to different sections of a quantum computer system. For instance, the protocols discussed in this paper could potentially be used to move states from memory to a system of quantum gates in an implementation of the circuit model.

There has already been significant work done on the subject of quantum data buses using spin chains, [7, 8, 9, 10]. However these works differ significantly from the method proposed in this paper in that the encoded q-bit is not transmitted through a degenerate ground state manifold, but through excitations of the Hamiltonian.

This paper investigates a method of using q-bits as an intermediate bus for the transfer of quantum information. This method can be compared to another method which is that of pulses [11], where a Hamiltonian is applied to a system for a period of time to perform a given operation. In the case of information transfer this operation is usually a swap. Unlike the method of using pulses, this method of using q-bits does not require precise timing to insure that the correct operation is performed. The method of using a spin chain Hamiltonian as a data bus also means that one does not need to either be able to address any pair of q-bits in the system or perform multiple operations to transfer an arbitrary q-bit. The pulse method does have the advantage that every intermediate spin can be used as quantum memory. However this is at the cost of the increased complexity of using dynamic quantum evolution in excited states, and the requirement of precise timing.

The adiabatic quantum bus method also has the advantage that, as in any adiabatic quantum process, only the lowest energy parts of Hamiltonian need to be faithfully realised by the implementation method. For example, a Hamiltonian which actually has an infinite number of excited states on each "spin", but where only the low energy states which act like a spin $\frac{1}{2}$ Heisenberg system, contribute to the ground state would be perfectly acceptable to use as an adiabatic quantum bus without modification. But the higher energy states may cause issues using a method such as pulses. This general feature of adiabatic quantum processes such as the one illustrated in this paper makes them more versatile than their non-adiabatic counterparts.

The effect we will examine exploits the SU(2) symmetry of the Heisenberg Hamiltonian and uses the ground state degeneracy created by this symmetry in a chain with an odd number of spins. It has already been demonstrated [12] that disturbances can be sent an unlimited distance along such chains because of their degenerate ground state. This paper goes a step further and actually demonstrates how a specific state can be transported across the chain using a quantum annealing protocol. Further investigation will also be provided into application of this method to systems such as the XYZ spin chain which only have a $\mathbb{Z}_2$ symmetry.

Setup

The model we consider is the J1-J2 Heisenberg spin chain with open boundaries,

$$H = \sum_{n=1}^{N-1} J_1^n \vec{\sigma}_n \cdot \vec{\sigma}_{n+1} + \sum_{n=1}^{N-2} J_2^n \vec{\sigma}_n \cdot \vec{\sigma}_{n+2}. \tag{1}$$

This model has SU(2) symmetry, which is expressed by the Hamiltonian

being block diagonal, such that there are N+1 blocks each with $\binom{N}{k}$ states. Each block represents all of the states with a given number, k, of up spins. If the number of spins in the model is odd, then the additional symmetry under a flip of the spins in the z direction, i.e. $\sigma^z \rightarrow -\sigma^z$ implies that all states of the Hamiltonian have a twofold energy degeneracy. In the anti-ferromagnetic case, ($J_1, J_2 > 0$) the ground state manifold consists of one state from the k=floor($\frac{N}{2}$) and one from the k=ceil($\frac{N}{2}$) sector. A simple example of this would be taking a system with 5 spins, the ground state would be twofold degenerate and would span the k=2 and k=3 sectors. One can now consider an initial Hamiltonian of the form of Eq. 1 where the couplings are the ones given by

$$
J_1^n = \begin{cases} J_1^{n,init} & n < N-1 \\ 0 & n = N-1 \end{cases},
\tag{2}
$$

$$
J_2^n = \begin{cases} J_2^{n,init} & n < N-2 \\ 0 & n = N-2 \end{cases}.
\tag{3}
$$

The general condition on $J_1^{n,init}$ and $J_2^{n,init}$ is that the coupling is predominantly anti-ferromagnetic everywhere and that each spin is coupled to the others by at least one non zero J. For simplicity this paper considers only $J_1^{n,init} = 1$ and $J_2^{n,init} = J_2^{init}$. This ground state manifold consists of the tensor product of the (unique) ground state of the chain of length N-1 with the Nth spin in an arbitrary state, a state in this manifold is of the from given by

$$
|\Psi^{init}\rangle = |\Psi_0^{N-1}\rangle \times |\psi^{init}\rangle,
\tag{4}
$$

where $|\Psi_0^{N-1}\rangle$ is the ground state of the spin chain of length N-1 and $|\psi^{init}\rangle$ is an arbitrary single spin state. One can now consider the same Hamiltonian, but with $n \rightarrow (N-n)+1$. This Hamiltonian also has the form of Eq. 1, but with couplings

$$
J_1^n = \begin{cases} J_1^{n,final} & n > 1 \\ 0 & n = 1 \end{cases},
\tag{5}
$$

$$
J_2^n = \begin{cases} J_2^{n,final} & n > 2 \\ 0 & n = 2 \end{cases}.
\tag{6}
$$

The general condition on $J_1^{n,final}$ and $J_2^{n,final}$ is that the coupling is predominantly anti-ferromagnetic everywhere and that each spin is coupled to the others by at least one non-zero J. For simplicity this paper considers only $J_1^{n,final} = 1$ and $J_2^{n,final} = J_2^{final}$. A state in the ground state manifold is now given by

$$
|\Psi^{final}\rangle = |\psi^{final}\rangle \times |\Psi_0^{N-1}\rangle,
\tag{7}
$$

where $|\psi^{final}\rangle$ is an arbitrary single spin state. One can now consider a quantum annealing process with described by

$$H(t;\tau) = A(t;\tau) \; H^{init} + B(t;\tau) \; H^{final},\tag{8}$$

where H^{init} is 1 with the conditions given in 2 and 3 and H^{final} is 1 with the conditions given in 5 and 6. Also A and B follow the conditions

$$A(t \leq 0;\tau) \;\; = \;\; 1,\tag{9}$$
$$B(t \leq 0;\tau) \;\; = \;\; 0,\tag{10}$$
$$A(t \geq \tau;\tau) \;\; = \;\; 0,\tag{11}$$
$$B(t \geq \tau;\tau) \;\; = \;\; 1.\tag{12}$$

For all values of A and B the SU(2) symmetry is preserved. Therefore the Hamiltonian remains block diagonal at all times. The symmetry of the Hamiltonian under $\sigma^z \rightarrow -\sigma^z$ is also preserved at all times. This implies that the ground-state degeneracy (as well as the twofold degeneracy of all states) is preserved. The block diagonal structure implies that there will be no exchange of amplitude between spin sectors during the annealing process, while the degeneracy implies that no relative phase can be acquired between the states in the k=floor$(\frac{N}{2})$ and the k=ceil$(\frac{N}{2})$ sector. From the combination of these two conditions one can see that as long as one anneals slowly enough with $H(t;\tau)$ [1] one can start with a state of the form given in Eq. 4 and reach a final state in the form Eq. 7 where $|\psi^{fin}\rangle = \exp(\imath\varphi)|\psi^{init}\rangle$, and φ is an irrelevant phase. One specific example of such an annealing protocol to transport a spin is given in Fig. 1.

Advantages

The use of the J1-J2 Heisenberg chain for transport by quantum annealing has several advantages. First the model with uniform coupling is gapped for $\frac{J_2}{J_1} \gtrsim 0.25$ [13]. This suggests that within the adiabatic evolution process, at least locally, the system should behave as a gapped system in this regime, as long as global effects such as odd length frustration do not cause problems.

It is important to note that even the largest system size considered here is far from the thermodynamic limit. One should note, however, that given the connectivity schemes of adiabatic quantum chips already in existence [5], one may only need to transport a q-bit state a few spins to get it to any part of the system.

Further evidence of favorable scaling comes from [12] which demonstrates that disturbances can travel an unlimited distance in the presence of a degenerate ground state, even in a gapped system. Furthermore, [12] suggests that these disturbances can carry entanglement, polarization, and quantum information. The transport by annealing given here is a specific example of how this effect can be taken advantage of.

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[1]Technically one must give the additional condition that there is no true crossing within the spin sectors on the annealing path.
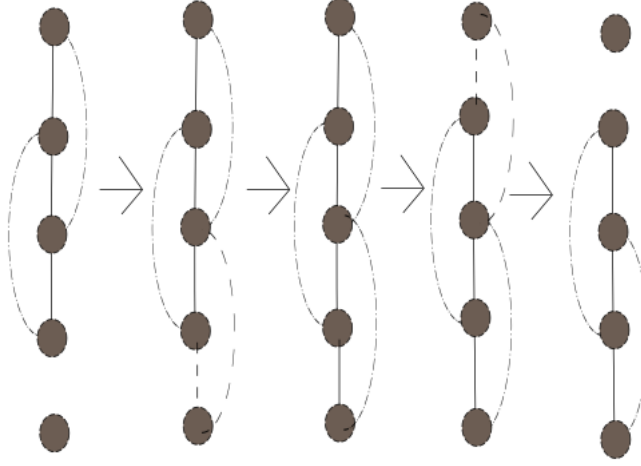
Figure 1: Cartoon representation of a process where a spin is joined to the chain, then the spin on the opposite end is removed. Note that this is only one specific example of many possible processes for transporting a q-bit.

Another advantage of the use of the J1-J2 Heisenberg Hamiltonian is the existence of the so called Majumdar-Ghosh point [14] ($\frac{J_2}{J_1} = 0.5$). At this point the ground state (with an even number of spins) has the simple form of a matrix product of singlets. Due to this fact the system should be relatively easy to prepare. The system is also gapped at the Majumdar-Ghosh point, making the Majumdar-Ghosh Heisenberg Hamiltonian, an ideal system for transport by quantum annealing and the ideal candidate for building an adiabatic quantum data bus.

Although this paper focuses on the J1-J2 Heisenberg model, it should be noted that this same annealing scheme should work with any pattern of coupling in the intermediate spins (i.e. J1-J2-J3)^2. One would also expect this scheme to work in models where the SU(2) symmetry is broken but there is a remaining $\mathbb{Z}_2$ symmetry such as the XYZ or XY model, again with arbitrary patterns of coupling. Note however that this mehhod will not work in the Ising model, because although there is a $\mathbb{Z}_2$ symmetry, the Hamiltonian lacks terms to exchange q-bits between sites because it is diagonal in the computational basis.

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^{2}At least this should work for small systems. In the continuum limit many of these systems may become gapless, so that quantum annealing cannot be effectively performed. Also one may be able to construct certain pathological cases with paths which pass through true crossings.

1 Proof of Principle

None of the arguments so far have given much illumination to the difficulty or ease of annealing within the sector. While we have discussed that transport of a q-bit state is possible in principle by annealing, we have not yet shown that the annealing process is fast enough to be practical. For this we turn to numerics. For the purposes of this paper we will consider the annealing time, τ, required to reach a given fixed fidelity, $F(\tau)$, with the true final ground state,

$$F(\tau) = |\left\langle \Psi^{fin} \mid \int_0^\tau dt\, H(t,\tau) \mid \Psi^{init} \right\rangle|. \tag{13}$$

The J1-J2 Heisenberg model is not an analytically solved model, at least for finite values of J_2, so numerical methods must be used in this calculation. One can first consider one part of the annealing process, in which a single spin is joined to a even length J1-J2 spin chain, using both J_1 and J_2 couplings which are linearly increased to equal values of those used in the rest of the chain [3],

$$H(t,\tau) = \sum_{n=1}^{N-2} J_1 \vec{\sigma}_n \cdot \vec{\sigma}_{n+1} + \sum_{n=1}^{N-3} J_2 \vec{\sigma}_n \cdot \vec{\sigma}_{n+2} + \lambda(t,\tau)(J_1 \vec{\sigma}_{N-1} \cdot \vec{\sigma}_N + J_2 \vec{\sigma}_{N-2} \cdot \vec{\sigma}_N), \tag{14}$$

$$\lambda(t,\tau) = \begin{cases} 0 & t \leq 0 \\ \frac{t}{\tau} & 0 < t < \tau \\ 1 & t \geq \tau \end{cases}.$$

As shown in Fig. 3, the annealing time required becomes large and highly sensitive to small variations for larger values of J_2. Also the behavior seems to get worse in this regime as system size is increased, and is poor at the Majumdar-Ghosh point [4].

As a further demonstration of the scaling with annealing time versus J_2, one can plot the annealing time versus system size, as we have done in Fig. 4. This figure shows polynomial or even sub polynomial scaling for small values of J_2, but than shows strongly non-monotonic behavior for stronger coupling. It is important to note however that even the longest chain length considered here is probably far from the infinite system limit, and this data may not be trustworthy for making predictions for scaling as the chain length approaches the infinite system limit.

By examining the gap one can hope to gain insight into the underlying cause of the behaviour of annealing time curves. As Figs. 5(a) and (b) show,

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[3]Note that this Hamiltonian (and all other annealing Hamiltonians in this paper) can be rewritten in the form of 8. However it is much more compact not to write the unchanging parts of the Hamiltonian twice.

[4]At least for fixed coupling, the case of dynamically changing coupling will be considered later.
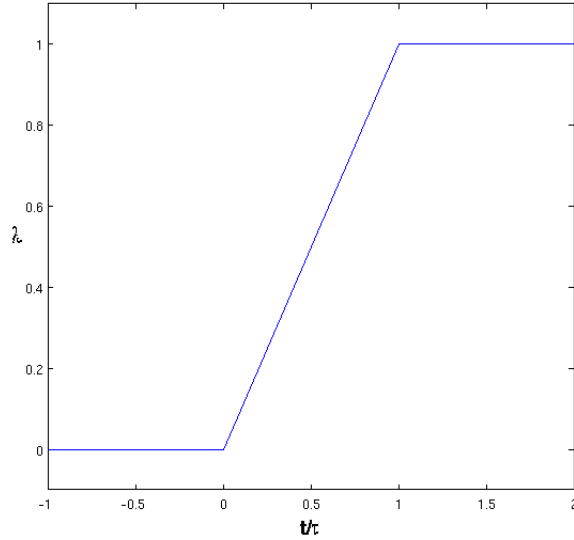
Figure 2: Coupling constant $\lambda(t, \tau)$ from Eq.14 and Eq. 16 versus $\frac{t}{\tau}$.

the behavior of the annealing time curves is reflected by the presence of what appear to be true crossings [5] for the odd length spin chain with uniform coupling. Fig. 5(b) shows the gap for an odd length spin chain and seems to confirm the presence of points with very small gap with uniform coupling for J_2 above 0.5. Figs. 3 and 5 together show that, at least at the length scales considered here, there are good annealing paths for joining a single spin to an even length chain. However, the simplest method of taking advantage of the simple ground-state wavefunction at the Majumdar-Ghosh point is not optimal. Fortunately there are many other possible options to take advantage of the easily prepared ground state and hopefully avoid the regions of small gap found here.

2   Dynamically Tuning J2

One method to avoid regions of small gap while still taking advantage of the Majumda-Ghosh point would be to start at the Majumdar-Ghosh point and then dynamically reduce the value of J_2 during the annealing process, a simple way of doing this would be to use the Hamiltonian in Eq. 15.

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[5]Strictly speaking nothing in this paper has demonstrated them to be true crossings, they could just be close avoided crossings, it does not matter for the purposes of this paper.
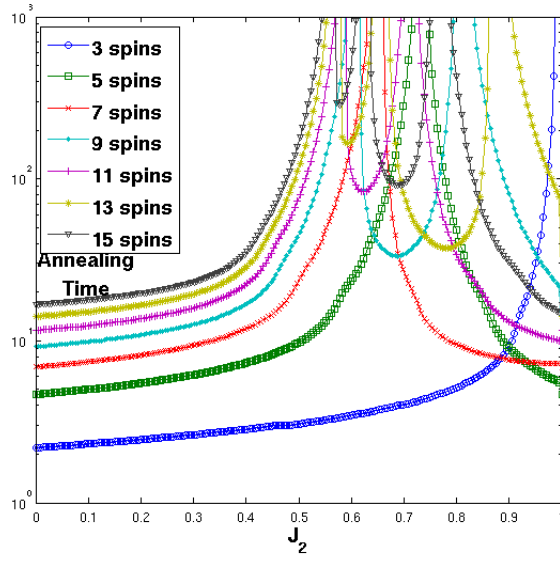
Figure 3: Annealing time required to reach a 90% fidelity with the true ground state within one of the two largest spin sectors of the Hamiltonian vs. J_2, with J_1 set to unity. One can see that for larger values of J_2 the annealing time behaves unpredictably. The annealing time also scales poorly with system size close to the Majumdar-Ghosh point.
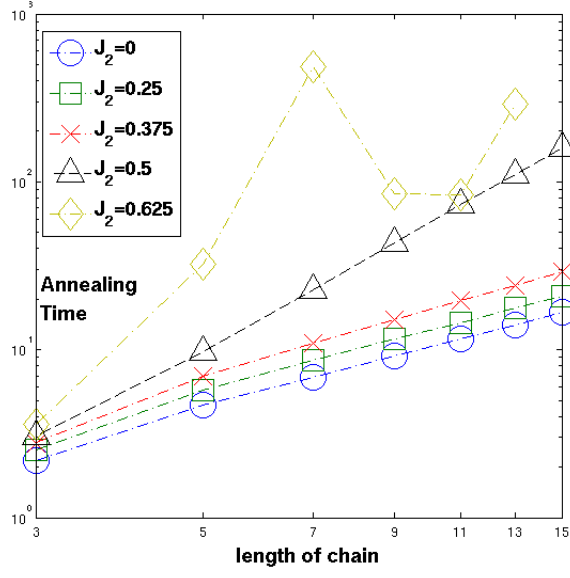
Figure 4: Scaling of annealing time to achieve 90% final ground state fidelity (in units of inverse Hamiltonian energy) versus length of chain on a log-log plot.
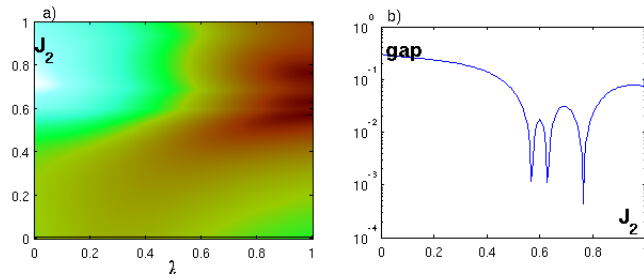
Figure 5: Plots of gap for joining a single spin to an even length J1-J2 Heisenberg spin chain. For density plots lighter colors indicate larger gap. a) gap versus λ in Eq. 14 and J_2 for 15 total spins d) Gap versus J_2 with $\lambda = 1$
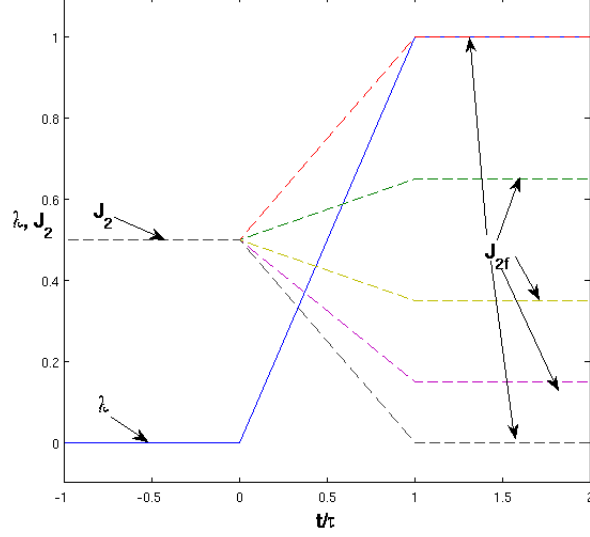
Figure 6: In this annealing protocol not only is a spin coupled to the chain, but J_2 is also changed dynamically.

$$H(t,\tau) = \sum_{n=1}^{N-2} J_1 \vec{\sigma}_n \cdot \vec{\sigma}_{n+1} + \sum_{n=1}^{N-3} J_2(t,\tau) \vec{\sigma}_n \cdot \vec{\sigma}_{n+2} + \lambda(t,\tau)(J_1 \vec{\sigma}_{N-1} \cdot \vec{\sigma}_N + J_2(t,\tau) \vec{\sigma}_{N-2} \cdot \vec{\sigma}_N),$$
$$(15)$$

$$\lambda(t,\tau) = \begin{cases} 0 & t \leq 0 \\ \frac{t}{\tau} & 0 < t < \tau \\ 1 & t \geq \tau \end{cases},$$

$$J_2(t,\tau) = \begin{cases} 0.5 & t \leq 0 \\ 0.5 + \frac{t}{\tau}(J_{2f} - 0.5) & 0 < t < \tau \\ J_{2f} & t \geq \tau \end{cases}.$$

Fig. 7 shows that taking advantage of the easily prepared ground state at the Majumdar-Ghosh point does in fact work, and the curves in this figure are strikingly similar to those in Fig. 3. This similarity is to be expected because Fig. 5 demonstrates that the gap is the smallest where the spin is completely joined. Hence this part of the process should dominate the annealing time.

It is reasonable to argue that because the regions of phase space which are visited are the same in the uncoupling process as coupling, the behavior of the system during the uncoupling process is determined by the gaps shown in Fig. 5,
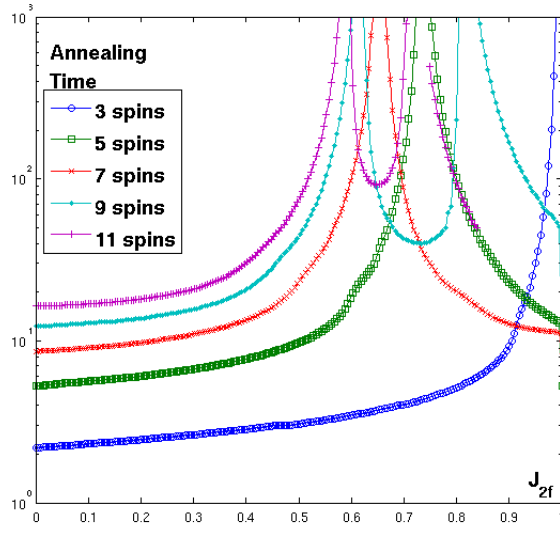
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Figure 7: Annealing time required to reach a 90% fidelity with the true ground state within one of the two largest spin sectors of the Hamiltonian with dynamical coupling starting at J_2=0.5 and linearly changing to J_{2f} while also joining a spin to the chain, with J_1 set to unity throughout the process. Notice that this figure is qualitatively and quantitatively very similar to Fig. 3.
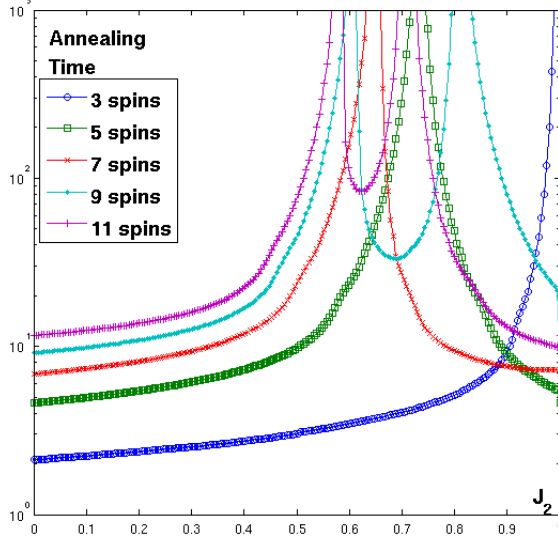
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Figure 8: Annealing time required to reach a 90% Fidelity with the true ground state for uncoupling process within one of the two largest spin sectors of the Hamiltonian vs. J_2 with J_1 set to unity. One can see that this figure is very similar to Fig. 3 as one would expect because it is simply the time reversed version of that process.

and therefore the annealing times for the uncoupling process should be at least qualitatively similar to those given in Fig. 3. One advantage to the uncoupling process is that unlike the coupling process, the need is not as strong to end in an easily prepared state. The only reason one may have to want to end in the Majumdar-Ghosh point is as an error check. The spins in the chain can be measured after the end of the process to ensure that no error has occurred [6].

Fig. 8 shows the time required to uncouple a spin from the chain, not surprisingly this figure looks very similar to Fig. 3 which is the coupling process. Note that in this system the Hamiltonian is simply Eq. 14 with $\frac{t}{\tau} \rightarrow (1 - \frac{t}{\tau})$.

As expected, except for one curve where a numerical error made some points unable to plot one can see from Fig. 9 that the uncoupling process also requires roughly the same time as the coupling process for dynamically tuned J_2. Note that the Hamiltonian for this process is simply Eq. 15 with $\frac{t}{\tau} \rightarrow (1 - \frac{t}{\tau})$ and $J_{2f} \rightarrow J_{2i}$.

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[6]For example if two spins which should be in a singlet together ended up being measured to be facing in the same direction than the annealing process would have failed.
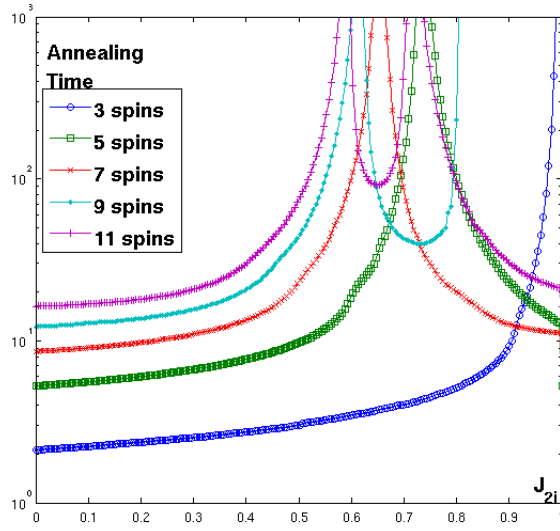
Figure 9: Annealing time required to reach a 90% fidelity with the true ground state for uncoupling process within one of the two largest spin sectors of the Hamiltonian vs. initial J_{2i} with a final J_2 at the Majumdar-Ghosh point with J_1 set to unity. This figure is very similar to Fig. 7 as one would expect, because it is simply the time reversed version of that process.
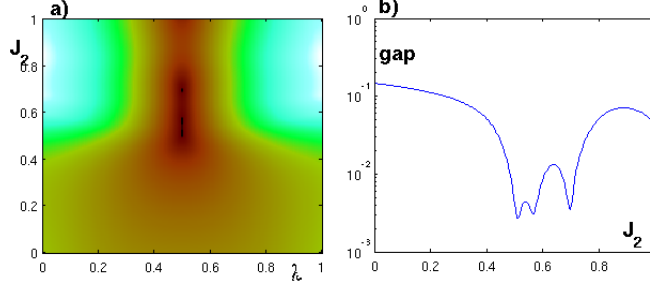
Figure 10: Plots of gap for simultaneously joining a single spin to an even length J1-J2 Heisenberg spin chain and unjoining a spin from the other end. For density plots lighter colors indicate larger gap. a) gap versus λ from Eq. 16 and J2 for 17 total spins b) Gap versus J_2 with $\lambda = 0.5$.

3 Simultaneous Uncoupling and Coupling

Because many of the issues encountered with the coupling protocol seem to relate to odd-spin frustration, it may be reasonable to consider simultaneously coupling one q-bit to the chain while uncoupling the other. The Hamiltonian in this case is given in Eq. 16.

$$H(t,\tau) = \sum_{n=1}^{N-2} J_1 \vec{\sigma}_n \cdot \vec{\sigma}_{n+1} + \tag{16}$$

$$\sum_{n=1}^{N-3} J_2(t,\tau)\vec{\sigma}_n \cdot \vec{\sigma}_{n+2} + \lambda(t,\tau)((J_1\vec{\sigma}_{N-1} \cdot \vec{\sigma}_N + J_2\vec{\sigma}_{N-2} \cdot \vec{\sigma}_N) - (J_1\vec{\sigma}_1 \cdot \vec{\sigma}_2 + J_2\vec{\sigma}_1 \cdot \vec{\sigma}_3)),$$

$$\lambda(t,\tau) = \begin{cases} 0 & t \leq 0 \\ \frac{t}{\tau} & 0 < t < \tau \\ 1 & t \geq \tau \end{cases}.$$

Fig. 10 shows the gaps for various system sizes for the process where the couplings are turned on and off simultaneously. This process does not seem to avoid the area of low gap for $J_2 \gtrsim 0.5$ seen in Fig. 5. However by comparing Fig. 10 d) and Fig. 5 d) one can see that it appears that the process of simultaneous uncoupling and coupling is characterized by avoided crossings rather than true crossings [7].

Fig. 11 shows the time required for annealing processes with for the combined coupling and uncoupling process, the results are consistent with what one

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[7]This statement is based on the fact that the gap does not have a cusp when plotted on a log scale. Strictly speaking this just shows that there is not a true crossing at the line where the two couplings are equal.
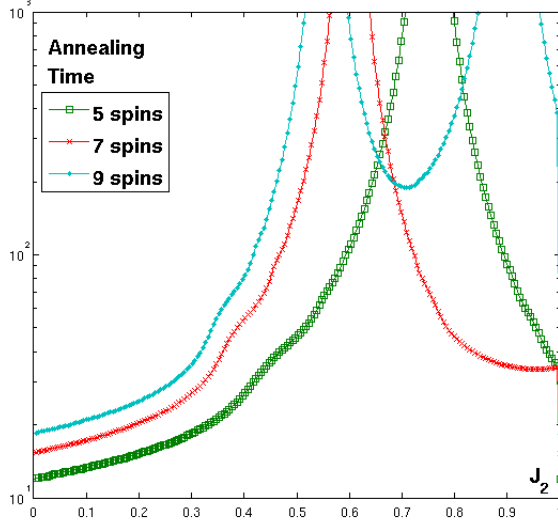
Figure 11: Annealing time required to reach a 90% Fidelity with the true ground state for combined coupling and uncoupling process within one of the two largest spin sectors of the Hamiltonian vs. J_2 with J_1 set to unity.

would expect from looking at Fig. 10, and confirm that the annealing time also tends to be very long and vary a lot for larger values of J_2.

4 Requirements for use as an Adiabatic Quantum Bus

It is now useful to consider a broader class of models that may be used as adiabatic quantum buses, as in general the full SU(2) symmetry of the Heisenberg Hamiltonian is not required.

The requirements for a spin chain (or network) Hamiltonain to be usable as an adiabatic quantum bus are as follow:

1. The ground state must be at least 2 fold degenerate, and the ground state manifold must be able to encode a q-bit. In this paper this is achieved by having at least a $\mathbb{Z}_2$ symmetry, and an odd number of spins, but there may be other ways.

2. The Hamiltonian (or at least the low energy states) must be predominantly anti-ferromagnetic in nature. This guarantees that the encoded q-bit will be excluded from the larger spin chain (or network) when a single q-bit is removed.

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3. The Hamiltonian must contain terms which perform exchanges between sites. This excludes models such as the Ising model which, although it has the required symmetry, cannot be used a quantum bus because its Hamiltonian is diagonal in the computational basis

4. One must be able to slowly couple in a spin with an arbitrary state on one end of the chain (network) and also to slowly remove coupling on the other end. More control may improve performance, but is not necessary.

5. Annealing paths in parameter space must not contain true crossings. This is a general requirement for adiabatic quantum computing.

5 XXZ and XYZ model

As previously mentioned, the full SU(2) symmetry of the Heisenberg Hamiltonian is not required. The Hamiltonian must only have a $\mathbb{Z}_2$symmetry to encode and transport one q-bit of information. In this section we will briefly examine two other possibilities: the XXZ model, where the SU(2) symmetry is broken, but the block diagonal structure imparted by this symmetry remains, and the XYZ model where only the block diagonal structure of a $\mathbb{Z}_2$ symmetry is present.

As one can see from Fig. 12, the XXZ model can be used as an adiabatic quantum data bus. There is a regime where this system outperforms the XXX Heisenberg model for Z/X between 1 and roughly 2. This is to be expected because adding additional coupling in the z direction may serve to open the gap between the the ground-state manifold and the next excited state. The increasing time as the z coupling is increased further can be explained because the system would behave like an Ising model in the limit of $\frac{Z}{X} \gg 1$.

One can further examine the behavior of an XYZ model as an adiabatic quantum spin bus. For this purpose we consider the quantum bus protocol performed on the following normalized XYZ Hamiltonian

$$H_{XYZ}(\Delta; N) = C_\Delta \sum_{i=1}^{N-1} \sigma_i^x \sigma_{i+1}^x + (1+\Delta)\sigma_i^y \sigma_{i+1}^y + (1+2\Delta)\sigma_i^z \sigma_{i+1}^z, \quad (17)$$

where the normalization is

$$C_\Delta = \frac{\sqrt{3}}{\sqrt{1 + (1+\Delta)^2 + (1+2\Delta)^2}}.$$

One can now examine the performance of this Hamiltonian for different values of Δ, noting that $H_{XYZ}(0; N)$ is simply the J1 Heisenberg spin chain of length N.

As Fig. 13 shows, a slight advantage can be gained by using an XYZ model rather than a simple Heisenberg chain. Fig. 13 also seems to suggest that the benefit gained is relatively independent of chain length.
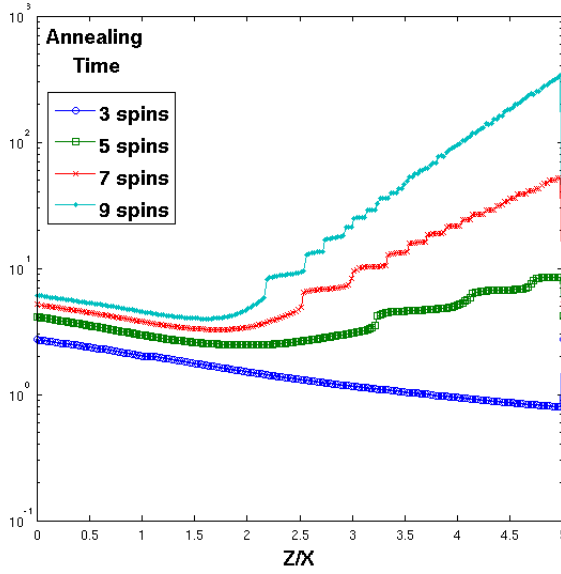
Figure 12: Annealing time to reach 90% fidelity on using the adiabatic quantum bus protocol on an XXZ spin chain versus the ratio of X and Z coupling strengths note that Z/X=0 is an XX model while Z/X=1 is a J1 Heisenberg spin chain. This data was obtained with joining and disconnecting of spins occurring simultaneously.
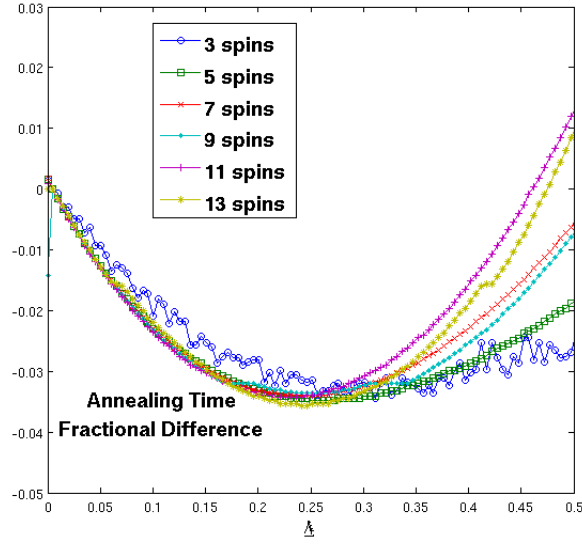
17

Figure 13: Plot of fractional difference from annealing time for an chain with small Δ (Heisenberg chain). This data is for the adiabatic quantum bus protocol performed on a chain of the form eq. 17 with spins being attached and removed simultaneously.

6 Other Protocols

So far we have only investigated a small subset of the possible annealing protocols which meet the criteria given in the introduction. For example the XY spin chain should also have and easily prepared ground state and may be easier to experimentally realize [1]. One could also try to examine the case of dynamically tuning the y and or z direction coupling and starting out at the Majumdar-Ghosh point but using modified coupling in the y and z directions with an XYZ model to avoid low gap regions.

One could also try to change the coupling scheme to avoid the low gap region, by either randomly or systematically modifying the coupling between intermediate spins, if this is done dynamically, one can still take advantage of the Majumdar-Ghosh point. This technique could also be used in conjunction with any of the ideas in the previous paragraph.

This paper is intended only to provide proof of principle for this method and is by no means an exhaustive search of all possible protocols.

7 Conclusions

We have demonstrated how a J1-J2 Heisenberg spin chain can be used to transport a q-bit state adiabatically. We have also shown that many extensions of this Hamiltonian; such as different coupling schemes or the XY or XYZ model which have only a $\mathbb{Z}_2$ symmetry, will also be able to be used to transport a q-bit [8]. We have found that for values of high frustration, transport by quantum annealing does not work very well. We have also demonstrated that this does not prevent us from exploiting the easily prepared ground state at the Majumdar-Ghosh point. We have given some examples of possible annealing protocols in this paper, but have really only investigated a very small section of a vast space of possible protocols for transportation of quantum states by annealing.

Acknowledgements

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